having arguments of the same period as that of the term he has deduced from the observation there are a number of others, such as the three depending on the arguments

$$\begin{aligned} &\mathrm{D} \quad -15\mathrm{V}+12\mathrm{E} \\ &2\mathrm{D}-4z+42\mathrm{V}-40\mathrm{E} \\ &\varpi+\ 2\mathrm{E}-\ 4\mathrm{M} \end{aligned}$$

The values of the coefficients of these terms as calculated by M. Radau's formulæ will be found to be much smaller than is required by Mr. Cowell; but this will be the case with the coefficient of every inequality of this period when so calculated. For the equation employed by M. Radau will not yield a coefficient exceeding a second of arc to any possible term of this period, unless values be assigned to Δb^{i} or ∇b^{i} respectively which are far beyond the maximum values of these functions.

Natal Observatory: 1904 *December* 9.

The Longitude of the Moon's Perigee. By P. H. Cowell.

In this paper I conclude my discussion of the Moon's observed longitude. I discuss the coefficients of $\sin g$ and $\cos g$, and I give values of the Moon's mean longitude and mean longitude of perigee.

To the values of the coefficients of $\sin g$, $\cos g$, derived from columns $53^{s}2$, $53^{c}2$ in Tables VIII., IX., *Monthly Notices*, vol. lxv. December, I have applied the following corrections, based upon the analyses exhibited in Table X. of that paper :

To the coefficients of $\sin g$—

(i.) $-0''\cdot7\cos(\omega-\omega')+0''\cdot2-0''\cdot7\sin(2\mathrm{D}-2g+3\mathrm{V}-3\mathrm{E})$

for the Airy period;

(iii.) $-0''\cdot7\cos(\omega-\omega')-0''\cdot2$

for the Hansen period.

To the coefficients of $\cos g$—

(i.) $-0''\cdot3\sin(\omega-\omega')-0''\cdot7\cos(2\mathrm{D}-2g+3\mathrm{V}-3\mathrm{E})$
$\qquad -1''\cdot0\sin(-\Omega)+2e\,\Delta g$

for the Airy period;

(iii.) $-0''\cdot7\sin(\omega-\omega')$

for the Hansen period.

Δg is the sum of (i.) the long-period *Venus* term; (ii.) Newcomb's empirical term with the same argument; (iii.) $-11''$.

The values of the corrected coefficients are exhibited in Table I.; the third columns refer to the Hansen period; periods 86–89 are common both to Airy and Hansen, and the results are enclosed in brackets, to call attention to this. In later stages of this paper the values taken for periods 86–89 are the means of the two sets of values here given, and the values for the 133 periods are then treated as continuous.

Table I.

Coefficients of sin g, cos g, corrected. Unit 0″·1.

Period.	sin g			cos g		
	0 +	48 +	85 +	0 +	48 +	85 +
+ 1	− 2	0	(−3)	+ 4	−10	(−10)
2	−10	0	(−5)	− 3	− 8	(− 5)
3	−10	0	(0)	+ 2	−14	(−18)
4	+ 4	+ 6	(+4)	− 3	−15	(−15)
5	+ 4	+ 9	+5	+ 6	+ 1	− 5
6	+ 6	− 5	+1	+ 3	− 6	− 9
7	+ 4	− 6	+2	− 1	− 5	− 1
8	− 4	+ 6	−7	− 1	− 7	0
9	− 4	− 5	−1	+ 6	− 8	− 3
10	− 1	+ 4	−5	+12	+ 7	− 4
11	−17	− 2	−5	+ 1	+ 3	−12
12	+ 7	+ 2	0	−13	− 3	− 5
13	− 7	− 4	+3	−14	+13	− 6
14	+ 1	− 7	+1	−12	− 4	− 5
15	+ 1	− 2	−3	− 6	0	+ 5
16	+ 7	0	−4	+ 4	+ 6	− 3
17	− 5	− 3	−3	0	− 2	− 6
18	− 1	+ 2	−2	−10	+ 4	− 5
19	− 5	− 9	+2	+ 2	− 1	− 5
20	+ 5	− 1	+3	+ 7	+ 1	− 4
21	− 2	− 2	0	+ 6	+ 1	0
22	+ 2	− 2	0	− 6	− 3	+ 3
23	− 1	+ 1	+2	− 9	+11	+ 1
24	+ 6	− 2	+1	− 6	+ 3	− 7
25	+ 9	− 9	−1	− 6	−10	− 3

Period.	sin g 0 +	48 +	85 +	cos g 0 +	48 +	85 +
+26	+14	+ 7	+2	+ 8	+ 1	+ 5
27	− 6	+12	+6	+26	+ 1	+ 2
28	− 1	+19	+1	− 3	+ 4	+ 5
29	− 2	+ 4	+4	+ 4	+11	+ 8
30	+ 3	0	+5	− 3	0	+ 8
31	− 1	+ 2	−7	+ 1	+ 2	+ 7
32	− 2	0	−3	−17	− 9	0
33	− 2	+ 2	+1	+ 6	+ 7	+ 5
34	+17	− 3	−5	−16	+ 9	− 9
35	− 2	− 6	+2	− 9	− 5	−11
36	− 4	− 1	+6	− 1	0	0
37	− 9	− 6	+4	− 7	+ 3	− 1
38	+ 4	(− 4)	−1	+ 4	(− 9)	− 3
39	+ 9	(− 6)	+3	+ 1	(−11)	0
40	− 5	(− 6)	−3	0	(−15)	+ 6
41	−11	(+ 5)	−4	−12	(−13)	+ 4
42	− 7		−1	− 1		+ 4
43	+ 4		−3	− 2		+ 1
44	− 2		+3	−11		+ 1
45	+ 8		−5	− 9		+ 4
46	+12		−3	− 5		+ 1
47	+11		−2	+15		+ 3
48	− 3		−6	+ 2		− 7

It will be noticed that the coefficients of $\sin g$ for the Hansen period are an extremely satisfactory set of numbers, and that they compare favourably with most of the columns of Table VIII., *Monthly Notices*, vol. lxv. December. The values of $\cos g$ are not so accordant. The conclusion, therefore, is that the outstanding corrections required fall into pairs of the form $\sin(g+\phi)-\sin(g-\phi)$ coalescing into $\sin\phi\cos g$; and that the coefficients of $\sin g$ conceal no periodic term with a coefficient greater than $0''\cdot 3$.

To analyse the numbers of Table I. I made use of the methods developed in my paper, *Monthly Notices*, vol. lxv. November. I took sums for 17 consecutive periods (for periods 7 and 127 I took sums for 13 consecutive periods only and then multiplied by $17 \div 13$) and added and subtracted quantities equidistant from period 67.

I thus obtained, in units of $1'' \div 340$, quantities for even and odd function analysis, which I exhibit in Table II.

TABLE II.

Coefficients of sin g and cos g arranged for Even and Odd Function Analysis. Unit $1'' \div 340$.

αt.	sin g Even.	sin g Odd.	cos g Even.	cos g Odd.
0	−38	...	+ 42	...
1	−11	+45	+ 42	+16
2	− 1	+27	− 8	+54
3	+14	+ 6	− 29	+49
4	+57	−17	−111	+33
5	− 6	−46	−142	+16
6	−23	−33	−167	−33
7	− 3	−43	−160	−34
8	−33	− 3	− 89	−31
9	+23	−41	− 78	−24
10	+46	−18	− 16	+14
11	+32	−14	+ 20	+ 2
12	+25	+ 9	− 18	+38
13	−10	+30	− 4	+64
14	−31	+ 9	− 7	+41
15	−55	+23	+ 16	+18

Although there is the appearance of a run in Table II. in the coefficients of sin g, the quantities involved are exceedingly small (about 0″·1).

I have calculated the quantities x defined in my paper, lxv. November, with the following results:

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
sin g unit $1'' \div 340$	−12	−25	+69	− 68	−7	− 26	−33	+ 17	+34
cos g unit $1'' \div 340$	+53	+82	+37	+102	+3	−171	−94	+154	+58
Δg unit $1'' \div 20$	+28	+43	+20	+ 54	+2	− 91	−50	+ 82	+31

A discordance between tables and observation in the coefficient of cos g implies a discordance nine times as great in the value of g. The third set of values given above has therefore been derived from the second by dividing by 17, to reduce the unit to 0″·05, and multiplying by 9.

At this point it is convenient to set down the tabular values of the mean longitude &c. that have been employed.

Airy's arguments are taken from Damoiseau's tables, 1824. I give them here reduced from Damoiseau's epoch, which Airy takes as 1801 January 0·5 − 9^m $21^s·5$ G.M.T. to Hansen's epoch of 1800 January 0·0 G.M.T. Next I copy Hansen's arguments g, g', ω, ω', $-\Omega$, and calculate from them L, L′.

Arg.	At 1800 Jan. 0·0 G.M.T.	Coefficient of T.	Coefficients of T².	T³.
Damoiseau.	° ′ ″	rev. ° ′ ″	″	″
L	335 43 28·70	1336 + 307 52 41·39	+ 10·723	+ 0·019361
g	110 19 47·63	1325 + 198 49 53·99	+ 50·418	+ 0·091035
$-\Omega$	326 43 49·40	5 + 134 9 57·63	− 6·563	− 0·011850
L′	279 54 40·45	100 + ... 45 53·00	...	...
g'	... 25 17·39	100 − ... 57 17·00	...	...
Hansen.				
g	110 19 33·64	1325 + 198 50 37·15	+ 49·435	+ 0·050073
g'	... 24 28·22	100 − ... 56 32·18	− 0·561	...
ω	192 7 21·91	16 + 243 12 2·07	− 44·323	− 0·043759
ω'	246 13 50·28	5 + 135 51 38·09	− 6·518	− 0·007159
$-\Omega$	326 43 28·85	5 + 134 8 59·61	− 8·189	− 0·007159
L	335 43 26·70	1336 + 307 53 39·61	+ 13·301	+ 0·013473
L′	279 54 49·65	100 + ... 46 6·30	+ 1·110	...
Hansen − Damoiseau.				
L	... − 2·00	+ ... ... 58·22	+ 2·578	− 0·005888
g	... − 13·99	+ ... ... 43·16	− 0·983	− 0·040962
L′	... + 9·20	+ ... ... 13·30	+ 1·110	...
g'	... − 49·17	+ ... ... 44·82	− 0·561	...
$-\Omega$	... − 20·55	− ... ... 58·02	− 1·626	+ 0·004691

To Hansen's tabular L Professor Newcomb has applied the correction in the *Nautical Almanac* since 1883

$$-1''·14 - 29''·17\ \mathrm{T} - 3''·76\ \mathrm{T}^2$$

I have applied the same correction to the individual tabular places from 1847 to 1882.

In my paper in *Monthly Notices*, vol. lxv. November, I have applied to the means for each period of analysis 1–89

$$-3''·14 + 28·63\ \mathrm{T} - 1''·1828\ \mathrm{T}^2.$$

This was intended to secure a uniform system of tabular places for the discussion of that paper; but it will be seen that owing to an error a correction $+0''·42$ T is still required by Airy's tabular places. As this correction only varies from $-0''·2$ to $+0''·2$ it is unimportant.

On certain assumptions as to the nature of the long-period empirical term required it appears from the opening paragraphs of the paper quoted that a correction

$$-4''\cdot3 \text{ or}$$
$$-4''\cdot3+5''\cdot2-3''\cdot0\ \mathrm{T}\ -4''\cdot4\ \mathrm{T}^2$$

is still required. Probably it is best to assume such a period for the empirical term as will make the secular acceleration equal to its theoretical value. The correction will then be about

$$-4''\cdot3+\tfrac{1}{2}\{+5''\cdot2-3\cdot0\ \mathrm{T}\ -4''\cdot4\ \mathrm{T}^2\}$$

to which we must add $+0''\cdot8$ if the formula is to represent the observations after 1902·0, which are now reduced to the epoch of Newcomb's catalogue. The mean longitude is therefore

$$335^\circ\ 43'\ 25''+(1336^{\mathrm{rev.}}+307^\circ\ 53'\ 9'')\mathrm{T}+7''\cdot3\ \mathrm{T}^2$$

subject to uncertainty of the order

$$\pm5''\pm3''\ \mathrm{T}$$

until theory has given a more precise value to the empirical terms.

Coming now to the mean anomaly, Professor Newcomb has applied the correction

$$-1''\cdot14-29''\cdot17\ \mathrm{T}-3''\cdot76\ \mathrm{T}^2$$

in the *Nautical Almanac* since 1883. I have applied the same correction to the individual tabular places from 1847 to 1882. I have also applied (see *Monthly Notices*, vol. lxiv. p. 85)

$$+4''\cdot63-13''\cdot99\ \mathrm{T}+4''\cdot743\ \mathrm{T}^2$$

to all tabular values of g based on Hansen. It will be seen that the value of g for the Hansen period has now been reduced to $-10''\cdot5$ (a constant) in excess of that used by Airy.

Owing to a numerical error, in the first table of this paper I have applied $-11''\cdot0$ to Airy's g; there is therefore a want of continuity of $0''\cdot5$ in g, or Airy's tabular places require a further correction of $+0''\cdot05 \cos g$. This is insensible.

In Table I. g also contains a long-period *Venus* term and Newcomb's empirical term of the same period. In my paper (*Monthly Notices*, vol. lxv. November) I came to the conclusion that Newcomb's empirical term had too short a period. Removing it therefore by applying to the x's (see *Monthly Notices*, vol. lxv. p. 51)

+532		−69		+2		−1		−2
	+316		−19		0		0	

we obtain for the x's of Δg

+560		−49		+4		−51		+29
	+359		+35		−91		+82	

If the value +359 of x_2 be attributed to a secular term in g we should obtain a concluded value entirely at variance with theory. Analogy with the long-period *Venus* term leads us to expect that the cause that produces the empirical term in the longitude will also produce the same term in the mean anomaly. I therefore apply the x's of my long-period empirical term (see *Monthly Notices*, vol. lxv. p. 51), which are:

−464		+32		0		0		0
	−237		+8		0		0	

obtaining for the corrected x's of g

+96		−17		+4		−51		+29
	+122		+43		−91		+82	

A better way of treating the subject is to form the x's of $L - g = \varpi$, the longitude of perigee. These are:

−86		+20		−35		+46		−13
	−86		+39		+21		−68	

It will be noticed that the effect of this last operation is identical with the effect of changing the sign of the row of x's immediately above, after adding in the x's of the two empirical terms (see *Monthly Notices*, vol. lxv. November) of 66 and 42 years' period.

As the x's for ϖ are smaller than the preceding set, to that very slight extent there is evidence from the observations for my two smaller empirical terms being accompanied by the terms formed by multiplying them by $2e \cos g$.

In the x's for ϖ I am obliged to dismiss as accidental all but $x_1 = -86$ and $x_2 = -86$. I can to some extent justify this by pointing out that the x's for $\sin g$ are probably accidental. For $\sin g$, $x_3 = +69$. Multiplying by the ratio 9 : 17 we get +37. I am now proposing to treat −68 the x_8 for ϖ as accidental. It is larger than I could wish, but I see no alternative. It must be admitted therefore that x_1 and x_2 are subject to possible errors ± 80.

The tabular longitude of perigee used in this discussion is Hansen's $\omega + \Omega$ increased by

$$-4''\cdot63 + 13''\cdot99\,T - 4''\cdot74\,T^2.$$

This requires correction by

$$+86\,t_1+86\,t_2$$

or by Constant $+2''\cdot7\,\mathrm{T}+7''\cdot4\,\mathrm{T}^2$, measuring T from 1826

or by Constant $-1''\cdot0\,\mathrm{T}+7''\cdot4\,\mathrm{T}^2$, measuring T from 1800.

To find the constant, the mean of the quantities in Table III. (*Monthly Notices*, vol. lxv. p. 39) is $-1''\cdot35$; the mean of the quantities in the last column of the first table of this paper is $-0''\cdot185$, corresponding to $\Delta g=-1''\cdot67$; the mean value of the correction to ϖ must therefore be $-0''\cdot32$; the constant is therefore $-3''\cdot7$, to which must be added $+0''\cdot8$ for the epoch of Newcomb's catalogue. The concluded value therefore exceeds Hansen's by

$$-7''\cdot5+13''\cdot0\,\mathrm{T}+2''\cdot7\,\mathrm{T}^2$$

or

$$\varpi=225^\circ\ 23'\ 46''+(11^{\mathrm{rev.}}+109^\circ\ 3'\ 15'')\,\mathrm{T}-33''\,\mathrm{T}^2$$

with possible errors of $3''$ in the mean motion for 1826 and $7''$ in the secular term, and perhaps $2''$ in the value for 1875.

Reducing the motion of the perigee to 1850 and subtracting $5024''$ for precession, I obtain for the observed sidereal motion of the perigee

$14643538''$ with a possible error of $\pm6''$.

Professor Brown (*Monthly Notices*, vol. lxiv. p. 532) quotes $14643523''$ as the observed value, which agrees more closely with his theoretical values than the observed value found by me.

My phrase "possible error" may be approximately taken to mean three times the probable error. I am assuming 20, 35, 46, 13, 39, 21, 68 to be accidental errors, and I am taking 80 as the measure of the "possible error."

As this paper concludes my discussion of the longitude I here give a summary of the results obtained.

1. Certain constants have been measured.

2. Table VIII. (*Monthly Notices*, vol. lxv. December) shows a close agreement between theory and observations as regards short-period terms. Probably there is no unknown term with coefficient $0''\cdot4$ or more having a mean motion differing by less than $0^\circ\cdot4$ a day from the mean motion of any one of the 40 auxiliary angles of that paper.

3. An apparent exception, depending on the argument $g+\omega-\omega'$ or possibly $g+\varpi$ is probably to be attributed to errors of north polar distance.

4. The coefficient ($6''\cdot6$) of sin ☊, the principal figure of Earth term, is decidedly smaller than the latest theoretical value. ($7''\cdot7$ G. W. Hill.)

5. Certain empirical terms have been obtained (*Monthly Notices*, vol. lxv. November).

The Final Values of the Coefficients in the New Lunar Theory.

By Ernest W. Brown, Sc.D., F.R.S.

1. As has been stated on previous occasions the problem under consideration, and now completed, is that of Delaunay's theory, with the additional terms introduced by replacing a/a' by $a(\mathrm{E}-\mathrm{M})/a'(\mathrm{E}+\mathrm{M})$. In earlier papers * I have given a general account of the methods used and of the means taken to secure accuracy, with some indication of the extent to which the results conform with those deduced from observation. The main object of the present communication is to give the complete numerical values of the coefficients of all periodic terms in longitude and latitude which are as great as $0''{\cdot}01$, and in parallax those which are as great as $0''{\cdot}001$. Every coefficient has been taken to at least one more place in the computations.

A secondary object is to compare the results with those of Hansen, so as to show explicitly the extent of the agreement between the two theories. The results of Delaunay may be used as a check where differences from those of Hansen occur; but slow convergence makes so many of Delaunay's coefficients doubtful that it does not seem useful to give them here. They can, if necessary, be directly obtained from Newcomb's "Transformation of Hansen's Theory," † the values which he there finds for the latter being inserted in the tables below. The general result of this comparison, stated briefly, is that where sensible differences occur between my results and those of Hansen my values are confirmed by Delaunay when allowance is made for the slow convergence of his series.

The mean motions of the perigee and node are not given here. Several comparisons with the results of Delaunay have been made on previous occasions, and the final numbers are fully set out and discussed in a paper "On the Degree of Accuracy of the New Lunar Theory and on the Final Values of the Mean Motions of the Perigee and Node." ‡

* In particular those in the *Monthly Notices* for April 1904 and December 1904.

† *Washington Astr. Papers*, vol. i. pt. (ii.)

‡ *Monthly Notices*, 1904 April, pp. 524–534. I am much indebted to Professor H. L. Rice of the Naval Observatory, Washington, for pointing out an error in the paragraph numbered 5 in this paper. The expressions

$$-\frac{3\delta a'}{a'}\cdot\frac{0''{\cdot}70}{328243},\quad +\frac{3\delta a'}{a'}\cdot\frac{0''{\cdot}20}{328243}$$

should be replaced by

$$-328243\,\frac{3\delta a'}{a'}\,0''{\cdot}70,\quad +328243\,\frac{3\delta a'}{a'}\,0''{\cdot}20,$$

and all the numbers in the second and third columns of the table headed "Indirect Planetary Action" (p. 529) should be multiplied by 1·077. Fortunately the changes in the final results are almost insensible: $-0''{\cdot}01$ is to be added to the calculated results for the perigee given on pp. 525, 532, and $+0''{\cdot}01$ to the calculated results for the node on the same pages.